Math 228: Solutions for Problem Set One

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1. **Page 23, number 10**

   (a) As you can see from below, the compared expressions have different truth tables, so they are not equivalent.

p	q	r	$q \to r$	$p \to q$	$(p \to q) \to r$	$p \to (q \to r)$
T	T	T	T	T	T	T
T	T	F	F	T	F	F
T	F	T	T	F	T	T
T	F	F	T	F	T	T
F	T	T	T	T	T	T
F	T	F	F	T	F	T
F	F	T	T	T	T	T
F	F	F	T	T	F	T

   (b) We already know that the two values are not the same. Therefore, the compound statement is not a truism. Therefore this is a counterexample to the associativity law in logic. This compound statement is also true whenever p is true or r is true so it is equivalent to $p \vee r$.

2. **Truth Table 2**

p	q	$p \to q$	$p \vee q$	$(p \to q) \to (p \vee q)$
T	T	T	T	T
T	F	F	F	T
F	T	T	F	F
F	F	T	F	F

   $(p \to q) \to (p \vee q)$ is independent of q and logically equivalent to p.

3. **Card flipping**

   To verify the statement, one has to flip any card showing a number besides 7 or showing a D.
   Proof A card is in one of four categories. Either the card is displaying a D or it is displaying a 7 or it is displaying a different letter or number.

   (a) The card is displaying a D.
   If the card has a 7 on the back, then it agrees with the statement. If the card

does not have a 7 on its back, then it provides a counterexample to the statement. The statement's truth depends on the card's value, so it must be checked.

(b) The card is displaying a 7.
If the card has a D on its back, it agrees with the statement that all cards with D's have 7's. If the card does not have a D on its back, it still agrees with the statement. So the value of the back of this card does not change the value of the statement. Therefore the back of this card does not need to be checked.

(c) The card is displaying a number besides 7
If such a card had a D on its back, it would be a counterexample to the statement. However, it supports the statement if it does not have a D on its back. Therefore these type of cards must be checked.

(d) The card is displaying a letter besides D
These cards cannot have a D on them, so they cannot provide a counterexample to the statement. They do not have to be checked.

Hence, the cards D, 2 and 9 must be checked.

4. **Knights and Knaves, Part 1**

Let's call B_n true if it goes to the capital. Because the natives consistently lie or tell the truth, all statements that a single native makes are logically equivalent.

$$\begin{aligned} &\text{From C: } B_1 \lor B_2 \leftrightarrow \neg B_3 \\ &\text{From D: } B_1 \leftrightarrow \neg B_2 \land \neg B_3 \\ &\text{From E: } \neg B_1 \leftrightarrow B_2 \land B_3 \end{aligned} \tag{1}$$

Suppose B_2 is true. Then $B_1 \lor B_2$ is true. From C, we know $\neg B_3$ must be true, which means B_3 is false. From E we have B_1 is true, but from D we have B_1 is false. This is a contradiction. So our supposition is false and B_2 is false.

If B_2 is false, then from E, B_1 is true. From D, B_3 is false. Thus B_1 is true, and B_2 and B_3 are false. So C and D are truthtellers and E is a knave.

5. **Knights and Knaves, Part 2**

If a computer has the internet, we'll call it true. Since a person speaking the truth makes true statements, they are logically equivalent to their own statements. Thus:

$$F \leftrightarrow \neg C_1 \leftrightarrow G \quad \text{and} \quad G \leftrightarrow C_2 \land \neg C_3 \tag{2}$$

$$H \leftrightarrow (C_2 \to C_1) \leftrightarrow \neg C_3 \iff \neg C_2 \lor C_1 \leftrightarrow \neg C_3 \iff C_2 \land \neg C_1 \leftrightarrow C_3 \tag{3}$$

Suppose C_1 is false. So $\neg C_1$ is true. Then from (2), $C_2 \land \neg C_3$ is true. From (3), since $\neg C_1$ is true, $C_2 \leftrightarrow C_3$. But if C_2 has the same value as C_3 then C_2 must have the opposite value as $\neg C_3$. So $C_2 \land \neg C_3$ cannot be true. This is a contradiction, so our supposition must be wrong and C_1 must be true.

If C_1 is true then $\neg C_1$ is false so (3) implies C_3 is false. But then $\neg C_3$ is true, so from (2), $C_2 \land \neg C_3$ is false, which implies C_2 is false. Hence, H is a knight and F and G are knaves.